# **Lesson Objectives**

1. The Basics of Linear Equations
2. Steps to solve a linear equation
3. How to deal with fractions
   * Find the Least Common Multiple (lcm) on calculator
4. Classify an equation as either conditional, identity, or contradiction
5. Problem Solving with Equations

# **Linear Equations – The Basics**

### How to **Determine** if an Equation is **Linear**

* It has only ONE variable (often *x*, but it could be *a*, *m*, *s*, etc.).
* The EXPONENT on the variable, wherever it is located, must ALWAYS be 1.

## **Basic terms** associated with a linear equation

* **Term** – a single number or variable, or numbers and variables mixed together. Terms in a linear equation are separated by the ADD or SUBTRACT sign.
* **Examples** of terms:
  + In the equation:
  + The terms are: and
  + Within the parentheses, there are also terms:
    - Within (9 – 7*x*), the terms are 9 and – 7*x*
    - Within (1 – *x*), the terms are 1 and *– x*
    - Within (*x* – 7), the terms are *x* and – 7
* **Coefficient** – the number to the immediate LEFT of a term containing variable.
  + The SIGN of the coefficient includes the add or the subtract symbol.
  + ADD means the term is POSITIVE.
  + SUBTRACT means the term is NEGATIVE.
  + If a variable has no visible coefficient, then it has an understood value of 1.
  + **Constant** – a term that has NO variable. It’s just a number of some kind.
  + Examples of terms (variable = V, constant = C) and their corresponding coefficients:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| term | – 2(9 – 7*x*) | – (1 – *x*) | 2(*x* – 7) | 9 | – 7*x* | 1 | – *x* | *x* | – 7 |
| type of term | V | V | V | C | V | C | V | V | C |
| coefficient | – 2 | – 1 | 2 | 9 | – 7 | 1 | – 1 | 1 | – 7 |

* The **Distributive Property** is used to “undo” or separate a coefficient next to parentheses.
  + – 2(9 – 7*x*) becomes – 2 · 9 + – 2 · – 7*x*, simplifying to – 18 + 14*x*
  + – (1 – *x*) or – 1(1 – 1*x*) becomes – 1 · 1 + – 1 · – 1*x*, simplifying to – 1 + *x*
  + 2(*x* – 7) or 2(1*x* – 7) becomes 2 · 1*x* + 2 · – 7, simplifying to 2*x* – 14
* **Like terms** – must contain the same type of variable(s), and same exponent(s)
  + Refer back to the original equation:
  + After the distributive property:
* **Combine (Add) Like Terms** – only done on the **SAME SIDE** of an equation.
  + NEVER combine like terms “ACROSS” an equation (from opposite sides)!
  + Left side: CONSTANT like terms – 18 and – 1, combine to make – 19.
  + Left side: VARIABLE like terms 14*x* and 1*x*, combine to make 15*x*.
  + Right side: NO like terms.

# **Steps to Solve a Linear Equation**

1. **\*\*Combine Like Terms**, if you can.
2. **Undo Parentheses**, using the Distributive Property, then **\*\*** (see #1).
3. (if necessary) **Clear out fractions** – multiply all terms by the common denominator (also known as the Least Common Multiple, or LCM), then **\*\*** (see #1).
4. **Letters go LEFT** – use ADD or SUBTRACT to move variable terms to the LEFT side of the equation, then **\*\*** (see #1).
5. **Numbers go RIGHT** – use ADD or SUBTRACT to move constant terms to the RIGHT side of the equation, then **\*\*** (see #1).
6. **Divide** – last step is to DIVIDE by the coefficient of your variable and simplify

So, returning to the **EXAMPLE** equation: [2.2.29]

You can’t combine like terms yet, so after Distributive Property:

Now you can Combine Like Terms:

Letters go LEFT:

updates to

Numbers go RIGHT:

updates to:

Last step, DIVIDE: updates to simplified:

# **How to Deal With Fractions**

* **EXAMPLE:** Solve the equation symbolically. [2.2-12]

A fraction means **DIVISION**, so first we need use **MULTIPLICATION** to undo fractions.

You need to multiply by the **least common multiple** of all the denominators.

We want the smallest multiple that is common for 2, 5, and 4.

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, **20**, 22, 24, ...

Multiples of 5: 5, 10, 15, **20**, 25, 30, …

Multiples of 4: 4, 8, 12, 16, **20**, 24, 28, …

So 20 is the smallest multiple found in all 3 lists (**Least Common Multiple**).

That process can sometimes take a long time, so here’s how it’s done on calculator:

## **Find Least Common Multiple (lcm) on Calculator.**

* Can only do 2 numbers at a time. If more than 2, “chain” together.
* No negative numbers. Just ignore the negative temporarily.
* No variables. Calculator can only do constants.

We need to find the Least Common Multiple (lcm) of 2, 5, and 4.

* **STEP 1:** Press **MATH**, move **right** to **NUM**, select **8: lcm(** 
* **STEP 2:** Enter **first number**, **comma**, **second number**, **close parentheses**, **ENTER**.

 

* **STEP 3:** If more than 2 numbers, take the answer and do lcm( again with 3rd number, etc.

 

Returning to the example problem – here it is written again:

* **EXAMPLE:** Solve the equation symbolically. [2.2-12]

The least common multiple (lcm) of 2, 5, and 4 is 20, so we need to multiply both sides of the equation by 20. This is called the **Multiplication Property of Equality**.

Use the **Distributive Property** next.

Simplify – **Divide out common factors**.

Use the **Distributive Property** again.

Simplify.

**Combine like terms.**

**Numbers go right**. (Addition Property of Equality)

**Combine like terms.**

**Divide** by the coefficient**.**

**Simplify** (reduce fraction, if you can, or convert to decimal and round, if needed).

Refer to embedded videos to help you with fractions – you NEED to know how to do these!

# **Classify an Equation as Conditional, Identity, or Contradiction**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Conditional** | **Identity** | **Contradiction** |
| **What happens:** | Solve “regular” equation, like normal | Variables will drop out, leaving a **TRUE** equation. | Variables will drop out, leaving a **FALSE** equation |
| **Finished equation looks like:** (examples) | *x* = some number | 0 = 0 or 7 = 7 (etc.)  Both sides are **IDENTICAL**. | 0 = – 3 or 5 = 14 (etc.)  Each side is different. |
| **Solution (answer) format:** | *x* = *a*  where *a* is a real number | **All real numbers** or | **No Solution** |

* **EXAMPLE:** Solve the equation symbolically. Classify the equation as a contradiction, an identity, or a conditional equation. [2.2.51]

**Clear out fractions**. The least common multiple of 4 and 10 is 20.

Simplify – **Divide out Common Factors**.

Use the **Distributive Property**.

Simplify

**Letters go LEFT.**

You have a **TRUE** equation. This is an **IDENTITY**. The solution is **ALL REAL NUMBERS**.

(go on to the next page)

* **EXAMPLE:** Classify the equation as a contradiction, an identity, or a conditional equation.

[2.2-18]

Use the **Distributive Property** to undo parentheses.

Simplify.

Combine Like Terms.

Simplify.

This is a **FALSE** equation, so this is a **CONTRADICTION**. This has **NO SOLUTION**.

# **Problem Solving with Equations**

* **EXAMPLE:** A store is discounting all regularly priced items by 75%. [2.2-29]
  1. Find a function *f* that computes the sale price of an item having a regular price of *x*.
  2. If an item normally costs $109.45, what is its sale price? Round to the nearest cent.

**(solution)**

1. First, we need to identify our variables: *f*(*x*) = sale price *x* = regular price

Next, when something is *discounted*, it is *subtracted* from the regular price (*x*).

Discounted 75% means discounted 75% of the regular price = 0.75(*x*) = 0.75*x*

To find a function *f* that computes the sale price of an item having a regular price of *x*:

Sale price = Regular price – the Discount

*f*(*x*) = *x* – 0.75*x*

The function is: ***f*(*x*) = *x* – 0.75*x***

1. If an item that normally costs $109.45, that means *x* = 109.45

Use the function *f* to find the sale price: *f*(*x*) = *x* – 0.75*x*

Evaluate (plug in) the function for *x* = 109.45: *f*(109.45) = 109.45 – 0.75(109.45)

= 27.3625 = **$27.36**

Sources Used:

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2. Pearson MyLab Math *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website <https://archive.codeplex.com/?p=wabbit>